NON-$\mathbb{Q}$-FACTORIAL DLT BIRATIONAL TRANSFORMATIONS

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0.1. **Non-$\mathbb{Q}$-factorial dlt birational modifications.** In this subsection, we give a remark on the log minimal model program for non-$\mathbb{Q}$-factorial dlt pairs. It heavily depends on [BCHM].

non-q-fac-dlt **Theorem 0.1.** *Let (X, Δ) be a dlt pair and $f : X \to Y$ a projective birational morphism between quasi-projective varieties. Assume that $-(K_X+\Delta)$ is f-ample. Then there exists a log terminal model (X', Δ') of (X, Δ) over Y. Moreover, we see that $K_{X'} + \Delta'$ is f'-semi-ample. In particular, we have the log canonical model of (X, Δ) over Y.*

Proof. Since $-(K_X + \Delta)$ is f-ample and Y is quasi-projective, we can write

$$\Delta - \varepsilon(K_X + \Delta) \sim_{\mathbb{R},f} \Theta$$

such that (X, Θ) is klt for $0 < \varepsilon \ll 1$ (cf. [KM, Proposition 2.43]). By [BCHM, Theorem C] (see also Theorem ??), we have a log terminal model $\phi : X \dashrightarrow X'$ over Y. Since $K_X + \Theta \sim_{\mathbb{R},f} (1 - \varepsilon)(K_X + \Delta)$, $\phi : X \dashrightarrow X'$ is also a log terminal model of the pair (X, Δ). By [BCHM, Theorem 3.9.1], $K_{X'} + \Delta'$ is f'-semi-ample. Therefore, the log canonical model of (X, Δ) over Y exists. □

By Theorem 0.1, Step ?? in ??[1] always works for dlt pairs.

References

bchm [BCHM]
km [KM]

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