ADDENDUM TO "ON ISOLATED LOG CANONICAL SINGULARITIES WITH INDEX ONE"

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Abstract. We add a supplementary argument to the paper: O. Fujino, On isolated log canonical singularities with index one.

In this short note, we will freely use the notation in [F]. As Masayuki Kawakita pointed out it, it does not seem to be obvious that the statement in Remark 5.3 in [F] directly follows from the proof of Theorem 5.2 in [F]. It is because $V'_1 \cap V'_2$ in Step 3 in the proof of Theorem 5.2 is not necessarily connected. Therefore, we would like to add the following proposition between Theorem 5.2 and Remark 5.3 in [F]. Note that the proof of Theorem 5.2 and Remark 5.3 in [F] are both correct. We just add a supplementary argument for the reader's convenience. We note that Remark 5.3 is indispensable for the proof of Theorem 5.5 in [F], where we prove that our invariant μ coincides with Ishii's Hodge theoretic invariant.

Proposition. *If $V'_1 \cap V'_2$ is disconnected, equivalently, has two connected components W'_1 and W'_2, in* Step 3 *in the proof of* Theorem 5.2*, then*

$$\mathbb{C} \simeq H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \xrightarrow{\delta|_{W'_i}} H^m(V', \mathcal{O}_{V'}) \simeq \mathbb{C}$$

is an isomorphism for $i = 1, 2$, where δ is the connecting homomorphism of the Mayer–Vietoris exact sequence.

Proof. We note that $H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \simeq \mathbb{C}$ for $i = 1, 2$ by Theorem 5.2. We also note that $H^m(V'_i, \mathcal{O}_{V'_i}) = 0$ for $i = 1, 2$ by Step 3 in the proof of Theorem 5.2. We consider the following Mayer–Vietoris exact sequence

$$\begin{aligned} \cdots \to\ & H^{m-1}(V'_1, \mathcal{O}_{V'_1}) \oplus H^{m-1}(V'_2, \mathcal{O}_{V'_2}) \\ \xrightarrow{\alpha}\ & H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \oplus H^{m-1}(W'_2, \mathcal{O}_{W'_2}) \\ \xrightarrow{\delta}\ & H^m(V', \mathcal{O}_{V'}) \to 0 \end{aligned}$$

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as in Step 3 in the proof of Theorem 5.2. Note that $\mathrm{Im}\alpha \simeq \mathrm{Ker}\delta$ is a one-dimensional $\mathbb{C}$-vector space. We consider the exact sequence:

$$\cdots \to H^{m-1}(V'_1, \mathcal{O}_{V'_1}) \to H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \to H^m(V'_1, \mathcal{O}_{V'_1}(-W'_i)) \to 0.$$

By the Serre duality,

$$H^m(V'_1, \mathcal{O}_{V'_1}(-W'_i))$$

is isomorphic to

$$H^0(V'_1, \mathcal{O}_{V'_1}(K_{V'_1} + W'_i))$$

for $i = 1, 2$. We can check that $H^0(V'_1, \mathcal{O}_{V'_1}(K_{V'_1} + W'_i)) = 0$ for $i = 1, 2$ by the same way as in Step 3 in the proof of Theorem 5.2. Therefore, the natural map, which is induced by the restriction,

$$H^{m-1}(V'_1, \mathcal{O}_{V'_1}) \to H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \simeq \mathbb{C}$$

is surjective for $i = 1, 2$. Thus, we see that

$$\mathrm{Im}\alpha \simeq \mathbb{C} \left(\subset H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \oplus H^{m-1}(W'_2, \mathcal{O}_{W'_2}) \simeq \mathbb{C}^2 \right)$$

contains neither $H^{m-1}(W'_1, \mathcal{O}_{W'_1}) \simeq \mathbb{C}$ nor $H^{m-1}(W'_2, \mathcal{O}_{W'_2}) \simeq \mathbb{C}$. This implies that

$$\mathbb{C} \simeq H^{m-1}(W'_i, \mathcal{O}_{W'_i}) \xrightarrow{\delta|_{W'_i}} H^m(V', \mathcal{O}_{V'}) \simeq \mathbb{C}$$

is non-trivial, equivalently, an isomorphism, for $i = 1, 2$. □

The statement in [F, Remark 5.3] follows from Step 3 in the proof of [F, Theorem 5.2] and Proposition.

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References

[F] O. Fujino, On isolated log canonical singularities with index one, J. Math. Sci. Univ. Tokyo **18** (2011), 299–323.

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