A MEMO ON "A CANONICAL BUNDLE FORMULA"BY O. FUJINO AND S. MORI

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In [FM, Section 3], we claim that $NL^{ss}_{X/C}$ is integral. Our claim in Section 3 in [FM] follows from the observations below. I think that this argument is slightly better than the original one.

1. Throughout this note, we consider the fiber space $f : X \to C$ such that C is a curve, X is smooth, $p_g(F) = 1$ and $\kappa(F) = 0$, where F is the generic fiber of f with $m = \dim F$.

2. For our purposes, we can assume that C is affine, $f : X \to C$ is smooth over $C^0 = C \setminus P$. We put $X^0 = f^{-1}(C^0)$, and consider the local system $R^m f_* \mathbb{C}_{X^0}$ on C^0 and the monodromy around P.

3. The key observation is the following one. Assume that the monodromy around $P \in C$ is unipotent. Then $L^{ss}_{X/C}$ is integral around P. See [F, Corollary 4.5].

4. We put $N = \mathrm{lcm}\{y \in \mathbb{Z}_{>0} \,|\, \varphi(y) \le B_m\}$, where φ is Euler's function and B_m is the m-th Betti number of F.

5. Let $f : X \to C$ be a given fiber space. When we take a unipotent reduction around $P \in C$, we can make the degree l of the base change $\pi : C' \to C$ satisfy that l divides N. We note that $\pi^* L^{ss}_{X/C} = L^{ss}_{X'/C'}$ for any finite morphism π, where X' is a resolution of $X \times_C C'$ (cf. [FM, Corollary 2.5 (ii)]). By the theory of the canonical extensions of Hodge bundles, we will prove that $L^{ss}_{X'/C'} = \llcorner L_{X'/C'} \lrcorner$ by the unipotency of the monodromy in 6 below. In particular, $L^{ss}_{X'/C'}$ is integral. Since $\deg \pi$ divides N, $NL^{ss}_{X/C}$ is integral because $L^{ss}_{X'/C'} = \llcorner L_{X'/C'} \lrcorner$ is integral.

6. Therefore, it is sufficient to prove that $L^{ss}_{X/C} = \llcorner L_{X/C} \lrcorner$ when the monodromy is unipotent. Let $\pi : C' \to C$ be a finite cover such that there exists a semi-stable resolution over C'. Then $\mathcal{O}_{X'}(\pi^* \llcorner L_{X/C} \lrcorner) = \pi^* f_* \mathcal{O}_X(K_{X/C}) = f'_* \mathcal{O}_{X'}(K_{X'/C'}) = \mathcal{O}_{X'}(\pi^* L^{ss}_{X/C})$, where $f' : X' \to C'$ is the semi-stable resolution. Here, we used the theory of the canonical extensions of Hodge bundles and the assumption on the monodromy

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I write this note because I am sometimes asked questions on [FM, Section 3].

to obtain the equality in the middle. We note that $\pi^* L^{ss}_{X/C} = L^{ss}_{X'/C'}$ is integral because $f' : X' \to C'$ is semi-stable. Thus, $\llcorner L_{X/C} \lrcorner = L^{ss}_{X/C}$.

7. We add one general remark. When we calculate discrepancies of $K_X + \Delta$, we have to fix a linear equivalence class of K_X. Similarly, when we consider $L_{X/C}$, its pull-backs, and so on, we fix one linear equivalence class of $L_{X/C}$ throughout the arguments.

8. Finally, we give a remark on [FM, Section 4]. In [FM, 4.4], $g : Y \to X$ is a log resolution of (X, Δ). However, it is better to assume that g is a log resolution of $(X, \Delta - (1/b)B^{\Delta})$ for the proof of [FM, Theorem 4.8].

References

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